S282 Astronomy

Stellar populations – Part 2

Population II stars in the solar neighbourhood

Study time: 2 hours

Summary

In this extended spreadsheet activity you are supplied with data tables of observations of the radial velocities of several hundred stars within about 300 pc of the Sun. The activity is split into two parts. In the first part, *Stellar motion in the solar neighbourhood*, you measured the motion of stars in our region of the Galaxy. In this part of the activity you will consider evidence for different stellar populations and examine some of their characteristics.

You should have read to the end of Section 1.4 of *An Introduction to Galaxies and Cosmology* before starting this activity and completed Part 1, *Stellar motion in the solar neighbourhood*.

Learning outcomes

- Appreciate the characteristics of the various stellar populations in the Galaxy.
- Experience using and interpreting observational data (positions and velocities).
- Appreciate the dynamic state of the Galaxy and the motions of stars.
- Gain skills in using computational and analytical tools (in this instance, spreadsheets) to interpret astronomical data.

Introduction to the activity

You know already that most stars in the disc of the Milky Way move in orbits which are approximately circular. In the first part of this activity you assessed the radial velocities of stars in the three cardinal directions $(l, b) = (180°, 0°)$ $(90°, 0°)$ and $b = 90°$, given the symbols U, V, and W respectively.

Question 1

The part of the Galaxy in the direction $(l, b) = (0°, 0°)$ is the Galactic centre. What names are given to the three directions corresponding to $(l, b) = (180°, 0°)$, $(90°, 0°)$ and $b = 90°$?

In Part 1 of this activity you plotted histograms for each of the three velocities, and calculated the mean velocity, the standard deviation, and the standard error in the mean. Although we did not dwell on it at the time, the standard deviation is a measure of the spread of the data points about the mean.

Question 2

What is the standard deviation σ that was measured for the radial velocities in each of the three cardinal directions? (Go back to your calculations in Part 1 to answer this.)

The activity

Modelling the velocity distributions

It was noted in Part 1 of this extended activity that the velocity distributions were roughly bell-shaped. We will now put that claim to a more rigorous test by computing the shape of a special bell-shaped curve that is called a 'normal' or 'Gaussian' distribution. Many natural processes give can rise to a distribution of a variable that has this shape. The approach we'll take is to compare our observed distributions of components of velocity with Gaussian/normal curves that have the same mean and standard deviation as each of our three samples.

The general expression for a normal distribution $f(x)$ of some independent variable x is the nasty-looking equation (which you shouldn't expect to remember)

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(-(x-\bar{x})^2/2\sigma^2\right)}$$

As an example, we let x represent the lengths (in mm) of objects which are all similar to one another. A plot of this function is shown in Figure 1, where the mean $\bar{x}$ is 100 mm and the standard deviation σ is 10 mm. The normal distribution is a symmetrical bell-shaped curve that reaches a maximum at the mean (=average) of the distribution, i.e. where $x = \bar{x}$. Where $x = \bar{x} + 2\sigma$ and $x = \bar{x} - 2\sigma$, the function has diminished considerably. The curve indicates that if you had a sample of objects whose lengths differed from one another according to this normal distribution, then most would have lengths between about 90 and 110 mm, while a few would be shorter than 80 mm or longer than 120 mm. Also, the total area under this curve is 1.0 (in dimensionless units).

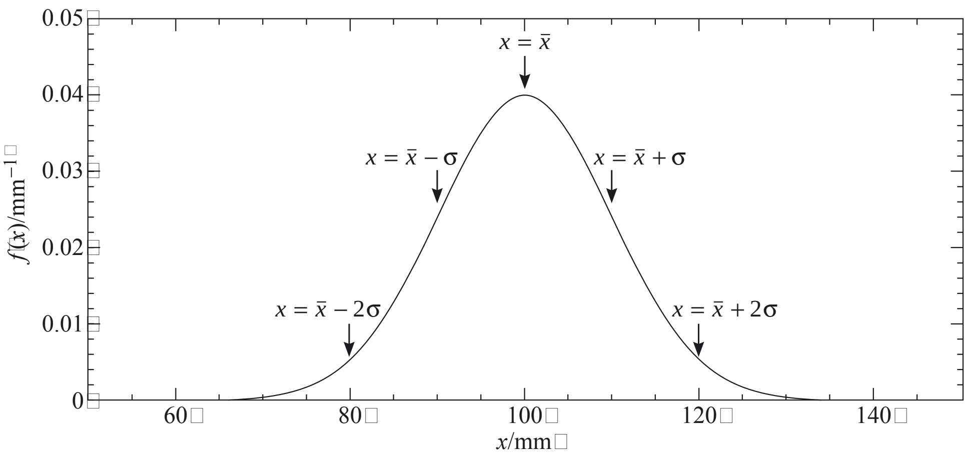

Figure 1 Normal distribution for $\bar{x}$ = 100 mm and σ=10 mm.

Begin by opening the spreadsheet that you completed in Part 1 of the activity. If you were unable to finish that activity, you might want to obtain the version of the final spreadsheet that has been supplied on the DVD.

Our aim is to plot a normal-distribution curve that can be compared with the observational data. To do this, we have to scale up the theoretical one, because the area under each of the three histograms calculated in Part 1 is more than 1.0. The area under a histogram of observational data is given by the number of stars in the data set, n, multiplied by the x-axis bin width, expressed in x-axis units. That is, since the x-axis bin width is 5 km s^{-1}, we multiply n by 5. That is, we need to rescale (multiply) the general formula for the normal distribution by $5n$.

Question 3

How many stars, n, are there in each of the three data sets?

The normal distribution can be readily computed for the data in the finished spreadsheet of Part 1 as follows.

- Begin with sheet U for the U-velocity, and put a heading 'normal' in cell Q1.
- Click in the next cell down, Q2, and prepare to type the formula for the scaled-up normal distribution.

What should that formula be? It will be easier if we work this out in pieces. From the expression for the normal distribution, we know we need an exponential term, $e^{\left(-(x-\bar{x})^2/2\sigma^2\right)}$. The exponential function is calculated within the spreadsheet using the built-in function `EXP( )`, and values are squared by following them with a carat symbol and the power two, `^2`. The velocity at the midpoint of each bin of the histogram is in column O, the mean velocity of the sample is in cell S1, and the standard deviation is in cell S2, so the exponential part of the formula can be typed

```
=EXP(0-(O2-$S$1)^2/(2*$S$2^2))
```

Note that dollar signs (`$`) have been used to freeze the cell references S1 and S2, so that the formula will pick up the entries for $\bar{x}$ and σ correctly when it is copied to other cells.

The other part of the equation for the normal distribution is the factor $\dfrac{1}{\sqrt{2\pi}\,\sigma}$.

The spreadsheet stores the value of π in the function `PI( )`, and the square root is calculated with the function `SQRT( )`, so we precede the exponential term by the factor

```
(1/(SQRT(2*PI())*$S$2))
```

Finally, we recall that we have to rescale the theoretical formula by a factor $5n$. Since the number of stars n is recorded in cell S3, we can enter the full formula in cell Q2 as

```
=5*$S$3*(1/(SQRT(2*PI())*$S$2))*EXP(0-(O2-
$S$1)^2/(2*$S$2^2))
```

- Having checked that this makes sense, copy (drag) the formula down to fill the column that is two cells to the right of each of the midpoint values, until you reach the cell on the same line as the last of the midpoint entries. This calculates the value of the rescaled normal distribution at each midpoint.

It is then a simple task to insert a new chart which plots both the histogram of stellar radial velocities and our approximate model of them, the normal distribution.

- Select (highlight) the three columns headed 'midpoint', 'frequency' and 'normal', and then clicking on Insert | Chart... to set up the new plot, following much the same process as in Part 1.
- Copy and paste the formula into each of the remaining sheets V and W, and produce new plots for all three velocities.

Here you see one of the beauties of working with spreadsheets; once you set up a formula, you can copy it not only to cells within the same sheet, but also to cells on other sheets which have the same layout (as ours do). If you wish to take a break, you may find this a convenient point at which to do so.

Interpreting the comparison

Now consider the histograms that you have plotted. Do you think these model distributions provide good fits to the observational data? (It would be possible to perform various statistical tests that would quantify how good the fits are, but that is unnecessary for the current purposes.) Consider each of the velocity components U, V, and W in turn, and reach an opinion about each one before proceeding.

The histograms for the U and W velocities look to be reasonable fits to the data, but it is clear that the V-velocity comparison is not very good. The modelled normal distribution is much too broad compared to the data, and could not be said to be a good fit. Assuming you have set up the equations correctly, and assuming the spreadsheet can handle the arithmetic correctly, what has gone wrong?

The fact that the calculated V-velocity distribution is too broad, and that the calculation of this distribution uses the value of the standard deviation σ_V that we calculated in Part 1, suggests that that value is not terribly useful. This is not to say it is wrong, but rather that it does not give a good representation of the data.

What is the difference between these two statements? The standard deviation indicates the spread of the data, that is for certain, but just because we can calculate a standard deviation does not mean the data conform to the normal distribution. The mismatch between the observations and the model curves for the V velocity is in fact telling us that the distribution is not normal. Your job as a scientist is to work out why, especially since the observed U and W velocities do give a reasonable match to the normal curve.

Inspection of the V-velocity histogram shows that even the calculated normal distribution, which we can see is too broad, is close to zero for $V < -100$ km s^{-1}. This suggests that there is something odd about the stars with velocities more negative than this. What might be wrong with these stars? A careful scientist might first enquire whether their radial velocity measurements were reliable. Once that is verified, the next step might be to ask whether these stars' velocities are meaningful. If, for example, these stars were members of binary systems so they were in orbit about a companion star, then their velocities would be telling us about the dynamics of their binary system, not about their Galactic motions. That is, the velocities would be accurate but inappropriate for our study. If, however, the stars are confirmed to be single stars, then we are left with a third conclusion, that their velocities are genuinely very different from those of other stars in the solar neighbourhood, and they are telling us about a system of stars other than the system which constitutes the local standard of rest. In fact, the last of these is the relevant conclusion in this case, and the five stars with $V < -100$ km s^{-1} are not part of the system of stars defining the local standard of rest. That is, they are not rotating around the Galaxy at roughly 220 km s^{-1} like most of the disc stars.

Question 4

If the stars with $V < -100$ km s^{-1} are not part of the system of stars defining the local standard of rest, what system might they be part of?

If the stars with large negative V velocities are correctly identified as not belonging to Pop. I, then we can use their properties to make rough estimates of the parameters of the population to which they do belong. A sort of the radial velocities (highlight column G and then click Data | Sort... | Ascending shows that the V values for these five stars are –314.3, –262.8, –166.2, –143.5, and –113.3 km s^{-1}. (Don't forget to do Edit | Undo sort to revert to the original file order.) If they belong to Pop. II, then we can use these values to estimate the rotation of Pop. II stars about the Galactic centre. This is the topic of Question 5.

Question 5

(a) By reusing the techniques you have used earlier in this activity to calculate the mean and standard deviation of a set of data, calculate the mean and standard error of the mean for the V velocities of the five Pop. II stars. (You may find it convenient to copy the V velocities for these five stars to a small table to the side of the existing tabulations in sheet V, and to make the calculation there. If you get stuck, look back at Part 1, in the section 'Interpreting the data', where means and standard deviations were first calculated. If you get *really* stuck, you can look into the solution spreadsheet for this activity which is supplied on DVD.)

(b) Assuming the Sun is overtaking the local standard of rest by 12 km s^{-1}, by what speed do Pop. II stars fall behind the LSR?

(c) Measurements of other astronomical objects suggest that the LSR moves around the Galactic centre at 220 km s^{-1}. Based on your measurements, at what speed do Pop. II stars orbit the Galactic centre (on average)? What does your result indicate?

(d) Use the data to estimate the approximate ratio of the number of Pop. II stars to the number of Pop. I stars in the solar neighbourhood.

Concluding remarks

Your analysis of the velocity distributions of the observed stars has led you to discern that a small proportion of the stars in the solar neighbourhood, about 1% of them, belong not to the disc population, Pop. I, but to the halo population, Pop. II. Furthermore, you have been able to estimate the mean rotation velocity of Pop. II stars as indistinguishable from zero, at 32 ± 38 km s^{-1}. This is in contrast to the rapid rotation of stars in the Galactic disc, which travel at about 220 km s^{-1}. To finish your analysis of the differences between the motions of Pop. I and Pop. II stars, consider the following question.

Question 6

(a) Comment on the standard deviation of the V velocity for the five halo stars compared to the overestimated value for disc stars that we calculated earlier.

(b) How might Pop. II stars be recognized by their velocities if they are observed not via radial velocities in the cardinal directions, but by transverse velocities in arbitrary directions?

Question 7

To complete your work on both parts of the extended activities on stellar populations, write a brief summary (approximately 250 words) of the observations you have used, and what you have shown about the stars and the stellar populations in the solar neighbourhood.

Answers to questions

Question 1

The directions $(l, b) = (180°, 0°)$ $(90°, 0°)$ and $b = 90°$ are, respectively, toward the Galactic anti-centre, Galactic rotation direction, and North Galactic Pole. (Refer back to Part 1 of the activity.)

Question 2

$\sigma_U = 31$ km s^{-1}, $\sigma_V = 29$ km s^{-1}, $\sigma_W = 22$ km s^{-1}.

Question 3

$n_U = 430$, $n_V = 445$, and $n_W = 420$.

Question 4

The stars with $V < -100$ km s^{-1} are being overtaken by the Sun by more than 100 km s^{-1}. One might reasonably suspect that they are really members of Population II rather than Population I.

Question 5

(a) For the five possible Pop. II stars, calculations of the mean velocity and its standard error give $\langle V \rangle = -200 \pm 38$ km s^{-1} (relative to the Sun).

(b) If the LSR is moving 12 km s^{-1} slower than the Sun, then the Pop. II stars are rotating $200(\pm 38) - 12$ km s^{-1} slower than the LSR, i.e. 188 ± 38 km s^{-1} slower than the LSR.

(c) If the LSR is rotating about the Galaxy at 220 km s^{-1}, then on average Pop. II stars must be rotating at $220 - 188(\pm 38)$ km s^{-1} $= 32 \pm 38$ km s^{-1}. This indicates that Pop. II stars have almost no net rotation about the Galaxy.

(d) If 5 out of the 445 stars in the Galactic rotation direction belong to Pop. II, then this suggests the fraction of Pop. II to Pop. I stars is $5/(445 - 5) = 0.01$. It is clear that Pop. II stars are quite rare.

Question 6

(a) The standard deviation of the V velocity for Pop. II stars is 85 km s^{-1}, which is considerably larger than even the overestimated value of 29 km s^{-1} that we derived earlier for the disc stars. This shows that Pop. II stars have much larger random motions than Pop. I stars.

(b) Because of the larger random motions of Pop. II stars, they could be observed as stars having apparently high velocities compared to most other disc stars even away from the cardinal directions. These are the so-called 'high-velocity' stars.

Question 7

We began in Part 1 with radial velocities for 420–450 stars in each of the three cardinal directions $(l, b) = (180°, 0°)$, $(90°, 0°)$, and $b = 90°$, located within about 300 pc of the Sun. By plotting velocity histograms, we found that most of these stars belong to the Galactic disc (Pop. I) and have velocities which differ from that of the Sun by no more than a few tens of km s^{-1}, with standard deviations in the range 22–31 km s^{-1}. We used the observed mean motions of the stars to infer the motion of the Sun relative to the local standard of rest. We found that the Sun is moving towards the Galactic centre at 3 km s^{-1}, is overtaking the motion of the local standard of rest by 13 km s^{-1} in the rotation direction, and moving towards the north Galactic pole at 10 km s^{-1}. The V-velocity distributions showed that about 1% of the stars in the solar neighbourhood belong not to the disc population, Pop. I, but to the halo population, Pop. II. The mean rotation velocity of Pop. II stars was found to be indistinguishable from zero, at 32 ± 38 km s^{-1}, in contrast to the rapid rotation of stars in the Galactic disc, which travel at about 220 km s^{-1}. The standard deviation of the V velocity for Pop. II stars is 85 km s^{-1}, which is considerably larger than even the value for the disc stars. Because of the larger random motions of Pop. II stars, they are sometimes recognized in the solar neighbourhood as 'high-velocity' stars.